Standards Coalitions Formation and Market Structure in Network Industries*

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Abstract

We discuss the formation of technical standards platforms in industries with network externalities where firms are free to choose their degree of technical compatibility with competitors. In our model, firms choose affiliation to a technical standards coalition in the first stage of a game, and play an oligopoly game in the second stage. In adding itself to a technical standards coalition, a firm benefits from the network effects of the whole coalition, but also faces increased competition in the output market from other firms in the coalition. Also, the increase of the size of the coalition changes the competitive position of members of that coalition relative to other firms. We find that the extent and size of coalitions at equilibrium depends crucially on the degree of the intensity of network effects. When network effects are very strong, full compatibility prevails. When externalities are slightly weaker, two standards coalitions are formed, a singleton, and one with all remaining firms. On the other extreme, for very weak network effects, the equilibrium is total incompatibility, and for slightly more intense network effects, coalitions are of small size. We characterize a number of other equilibria for intermediate strengths of network externalities.

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1. Introduction

The issue of choice of technical compatibility standards is the key to understanding strategic choices in network industries and the resulting equilibrium market structure. This paper establishes the choice of technical standards and the formation of technical standards associations in network industries.

The existence of network effects implies that the value of a good increases with the size of the sales of technically compatible products. A firm's adherence to the same technical standard (same platform) with other firms allows the firm to receive the network effects benefits of the total production of all firms in the same platform. At the same time, competition among firms in the same platform is expected to be more intense since their products are less differentiated. These two opposing incentives determine the direct benefits and costs to a firm of affiliation with other firms on the same technical standard. There are also indirect costs and benefits that come from the effects on the oligopoly outcome of the formation or expansion of technical standards associations. For example, one expects that the expansion of a standards association will diminish profits of firms outside it.

We model firms' choices as a two-stage game. In the first stage, firms choose whether to affiliate with other firms so as to share a technical standard (platform). The result of the first stage game is a partition of the set of firms into associations (coalitions), where the same technical standard prevails within each coalition, and different standards prevail across coalitions. The oligopoly of the second stage is specified so that competition is more intense among firms in the same coalition and less intense between firms that belong to different coalitions. It is also assumed that an increase in the size of a coalition is beneficial to all its members and detrimental to non-members. Finally, it is assumed that firms' profits decrease as firms become more concentrated in terms of compatibility.

We set the coalition formation (first) stage of the game following the models of Ray and Vohra (1997, 1999) and Bloch (1996, 1997). Firms are assigned an index and make sequential proposals starting with the firm of the smallest index. Firm 1 makes a proposal to specific firms to include them it its coalition. The offer consists of the set of firms that are invited to form a coalition and conditional statements that describe the division of the coalition's worth conditional on the final coalition structure of the industry. Firms respond in sequence. If all recipients of the proposal accept it, the specific coalition is formed, its members exit the game, and the next lowest index firm announces the next proposal. If one firm rejects the original offer, then this firm initiates a new proposal. We focus our attention on stationary equilibria of this game, i.e., equilibria in which the strategies depend only on payoff-relevant elements of the game: the coalitions that have already exited the game and the set of firms that still remain in the game. Following Ray and Vohra (1999) this game always has a stationary equilibrium where the only source of mixing is in the (possibly) probabilistic choice of a coalition by each proposer. These are the equilibria we characterize in this paper.[1]

The intensity of network effects plays a crucial role in the structure of equilibrium. For very weak network effects, the equilibrium is total incompatibility,

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[1] The Bloch (1996) model assumes that the coalition worth is divided according to a fixed rule. The equilibrium coalition structures that we study are the same in both scenarios. We come back to this issue in section 2.2.

where each firm has its own technical standard. A striking feature of the equilibrium is that, except for the total incompatibility case, it is characterized by very significant inequalities among the sizes of coalitions. That is, unless network externalities are very weak, the equilibrium does not have equal sizes coalitions at equilibrium. For very strong network effects, the equilibrium is full compatibility, with all firms sharing the same technical standard. For weaker network effects, one firm defects, and for even weaker, a second firm joins it in defection. Finally, for large number of firms and for intermediate levels of network effects, the game has no equilibria in pure strategies without delay.

2. Formation Of Standards With Cash Transfers

2.1 General Setup

Let an industry have N *ex-ante* homogenous firms. Before playing an oligopoly production game with other firms, each firm chooses a technical standard (platform). Each firm can only choose one such platform. All firms that choose the same platform receive the total network benefits of this platform. Formally, we will consider all firms that choose the same platform as belonging to the same coalition.

Denote by $\boldsymbol{C}$ the set of all possible partitions of N. For a coalition structure $C \in \boldsymbol{C}$, denote by $K(C)$ the number of standards (coalitions) in this structure. Denote by n_k the number of firms in structure k for $k = \{1, ..., K(C)\}$. Let $s_k = n_k/N$. Finally let $s_k(i)$ to be the market share (in terms of number of firms) of the coalition containing firm i.

We introduce the concept of the Herfindahl-Hirschman index for the coalition structure $HH(C)$, defined as

$$HH(C) = \sum_{k=1}^{K(C)} (s_k)^2 . \tag{1}$$

This index measures market concentration of platforms (coalitions). Note that this is not the standard definition of the Herfindahl-Hirschman index ("HHI") of market concentration. It differs in two respects. First, it defines concentration among platforms (coalitions) rather than concentration among firms. Second, because we do not have output data, the index is defined in terms of firms rather than in terms of firms' outputs. But the index *HH(C)* is similar to the traditional HHI in the sense that it ranges between zero and one, and it increases as the market gets more concentrated (here in terms of platforms). The more firms adhere to the same technical standard (belong to the same coalition), the higher is the value of *HH(C).* Also, *HH(C)* decreases as the sizes of the platforms (coalitions) come closer to equality.[2]

The firms play a two-stage game. In the first stage, firms decide what coalitions to form (and how to divide their profits): firms that are in one coalition adopt one common standard, but firms in different coalitions have incompatible standards. During the second stage firms compete in the market given the coalition structure.

There are three important factors that influence the outcome of the second stage. The first is the existence of network effects: those platforms with a larger number of firms are more valued by customers. The second factor that influences the equilibrium is competition among firms within the same standard. The third factor is the degree of competition across standards. We expect that competition among firms that adhere to different standards will be less fierce than competition among firms that adhere to the

[2] *HH(C)* also exhibits the other usual properties of HHI. For example, the combination (merger) of two coalitions of sizes s_i and s_j into a coalition of a single technical standard increases *HH(C)* by $2s_is_j$.

same standard. Firms in the same coalition are not differentiated by the compatibility standard and hence we assume that they compete more strongly than two firms in different coalitions. In general, the interaction between these three factors and their impact on the profits can be very complicated, and there are many ways that it can be modeled.

We propose the following simplified reduced form of profits for a given coalition structure. We believe that this form captures the main features described above. Specifically, let the gross profits (gross of transfer payments) of firm i in coalition structure C be strictly positive and a strictly increasing linear function[3] of

$$V_i(C) = \beta + \alpha s_k(i) - (1-\alpha)HH(C), \tag{2}$$

where $HH(C)$ is the Herfindahl-Hirschman index for the coalition structure as defined above, $\beta > 0$, and α, $1 > \alpha > 0$, measures the relative strength of the network effect compared with the competition effects.[4] The higher the value of α, the higher the influence on profits of the size of the coalition in which the firm belongs and the smaller the influence of the overall market conditions summarized by $HH(C)$. In one extreme, when $\alpha = 1$, all that matters to profits is the size of the coalition it belongs to. In the other extreme, the size of the own coalition does not matter at all except to the extent it influences the market index $HH(C)$.

This profit function is a reduced form of the oligopoly outcome of the second stage of the game and has the following three properties. First, for any coalition

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[3] The linearity of profits in V(C) is necessary for the analysis of transfers between firms, so that the total surplus of the coalition a function of the sum of $V_i(C)$ for firms belonging to this coalition. In case the firms do not use transfers and divide the surplus equally, the linearity assumption is not necessary.

[4] Instead of α and $1 - \alpha$, we could have alternatively used weights $b > 0$ and $c > 0$, and then normalized by dividing by $(b + c)$ and defining α as $\alpha = b/(b + c)$.

structure, firms in a larger coalition earn higher profits than firms in a smaller coalition, that is, $V_i(C)$ is an increasing function of $s_k(i)$. This is an immediate consequence of the network effect. More firms in the same coalition result in a higher network effect and this implies higher profits.

Second, profits of firm i in platform k are a decreasing function of the degree of concentration among the competing standards as summarized by $HH(C)$. For example, the adoption of a common standard by two coalitions that originally had different technical standards increases $HH(C)$ and therefore decreases profits of all firms which do not adhere to this common technical standard.

Third, a coalition considering adding one additional firm faces the following trade-off: adding it is beneficial because it increases the network effect, but is detrimental because it directly increases concentration. At the same time there is an indirect effect because increasing s_k may change the choices of coalition affiliations of the remaining firms.

2.2 <u>Coalition Formation</u>

We model the first stage of the game as in Ray and Vohra (1999). The formation of coalitions on technical platforms is modeled as an alternating-offers bargaining game, $\Gamma(\delta)$, where δ is the discount rate to be explained below.[5] Firms are assigned an arbitrary index (a protocol of the bargaining game). Firm number 1 proposes the creation of an association (coalition) S which includes itself and possibly some other members that will share the same technical standard. Along with the coalition, the proposer

[5] See Rubinstein (1982).

announces a feasible division of the worth of the association conditional on the final coalition structures. Prospective association members respond in sequence to the proposal. If all prospective association members accept the offer, then the coalition is formed, and the procedure starts again among the remaining firms, initiated by the firm of the lowest index among them. If all prospective association members do not accept the offer, then the first firm that rejected the proposal makes the next proposal of a coalition that includes itself. In addition, a rejection creates a delay in the bargaining and imposes a geometric cost on all players that is captured by a common discount factor $\delta \in (0, 1)$. The procedure continues until an equilibrium in reached. The payoffs of the firms in the final coalition structure are given by equation (2), the contract specifying the division of surplus and the number of times a proposal was rejected. If no agreement is ever reached, all players receive zero payoffs.[6]

For example, if there are three firms, firm 1 may propose the formation of one of the following coalitions: {1}, {1, 2}, or {1, 2, 3}. If it proposes {1, 2, 3} and firm 2 rejects the offer, firm 2 will have to make a proposal of its own. If firm 2 accepts the offer and firm 3 rejects it, firm 3 has to make a proposal of its own. If both 2 and 3 accept, the {1, 2, 3} coalition is formed. If firm 1 proposes {1, 2}, firm 2 may reject it and then make its own proposal. Alternatively, if firm 2 accepts the offer, firm 3 does not have a choice in the matter and the coalition structure {{1, 2}, {3}} is formed.

In this setup, firms make their decisions anticipating the subsequent decisions of other firms in forming coalitions. Thus, each firm explicitly takes into account the effect

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[6] Recall that the payoffs given any coalition structure a strictly positive and strictly increasing linear function of $V_i(C)$, so that any agreement strictly dominates infinite disagreement.

of its actions in either proposing or accepting an offer on the incentives of other firms to form coalitions.

2.3 <u>Existence And Uniqueness Of Stationary Equilibria</u>

In this game we focus our attention on stationary strategies, i.e. strategies that depend only on payoff-relevant components of the game: the structure of the coalitions that have already formed and on the set of players that are still in the game. We define a stationary (perfect) equilibrium as a collection of stationary strategies such that there is no history at which a player benefits from a deviation from her prescribed strategy. Ray and Vohra (1999) show in their Theorem 2.1:

There exists a stationary equilibrium of the game Γ where the only source of mixing is in the (possibly) probabilistic choice of a coalition by each proposer.[7]

From now on, we focus only on stationary equilibria of this game. Bloch (1996) and Ray and Vohra (1999) propose a simple algorithm for finding candidate equilibria. We follow the description of Bloch (1996). Define a game Δ of choice of coalition structures as follows. Player 1 starts the game and chooses an integer $n_1 \in \{1, ..., N\}$. If $n_1 = N$, the game ends. Otherwise, player $n_1 + 1$ chooses an integer $n_2 \in \{1, \ldots, N - n_1\}$, and so on, until the sum of integers chosen by players equals N. The payoffs are given by equation (2) and the coalition structure corresponding to the integers chosen by the players. Game Δ is a finite extensive form game with perfect information, so it always has a SPNE, which is generically unique. Adding an assumption that players indifferent

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7 Ray and Vohra (1999), p. 294.

over several integers always choose the largest one, we obtain uniqueness in the non-generic cases. We call $\boldsymbol{C^*}$ the unique equilibrium coalition structure of Δ.

We define a *no-delay equilibrium* (of the game $\Gamma(\delta)$) to be one in which, at every stage, every proposal that is made is accepted. The unique equilibrium coalition structure $\boldsymbol{C^*}$ of Δ in many cases corresponds to a no-delay equilibrium coalition structures of the game $\Gamma(\delta)$. In particular, Theorem 3.1 in Ray and Vohra (1999) states:

There exists a $\delta^* \in (0, 1)$ such that for all $\delta \in (\delta^*, 1)$, any stationary equilibrium of game $\Gamma(\delta)$ in which an acceptable proposal is made with positive probability at any stage must induce a coalition structure that is identical to $\boldsymbol{C}^*$ up to a permutation of firms. [8]

Thus, this theorem states that all equilibria with no-delay have to induce the equilibrium coalition structure $\boldsymbol{C}^*$. So the question is, under what conditions does the game $\Gamma(\delta)$ have such equilibria? This is an open question, but Ray and Vohra offer a partial answer. We paraphrase Theorem 3.2 Ray and Vohra (1999):

If the payoffs of the firms in game Δ are weakly decreasing in a firm's index, then there exists $\hat{\delta} \in (0,1)$ such that for all $\delta \in (\hat{\delta},1)$ there exists a stationary equilibrium of game $\Gamma(\delta)$ in which an acceptable proposal is made in every stage with probability 1. [9]

Moreover, weakly decreasing payoffs in a firm's index in game Δ is a necessary condition for existence of pure strategy no-delay equilibria in game $\Gamma(\delta)$ as Theorem 3.3 of Ray and Vohra (1999) shows. Finally, Theorem 3.4 of Ray and Vohra (1999) provides

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8 Ray and Vohra (1999), p. 297.

9 Ray and Vohra (1999), p. 299.

a technical sufficient condition that guarantees the uniqueness of the no delay equilibrium.

In many cases in the network industries markets that we study, the weak monotonicity of the payoffs as well as the technical condition mentioned above are satisfied, and the unique equilibrium of game Δ is also the unique pure strategy no-delay equilibrium in game $\Gamma(\delta)$ for sufficiently large δ. However, for some large N (starting at N = 14) and for intermediate values of α we have found cases that the monotonicity is violated. This implies that there does not exist a pure strategy no-delay equilibrium in the game Γ. It remains an open question if in these cases there exists a no-delay equilibrium in mixed strategies (in some cases certainly not – see section 2.10). And if not, do there exist stationary equilibria with strictly positive probability of delay that induce $\mathbf{C}^*$? We conjecture that yes, but we do not have a proof.

Given these results, we now turn to characterizing equilibrium coalition structures of the game $\Gamma(\delta)$. We discuss only the partition of the firms for high discount factors. We do not discuss the division of the surplus, because first, the order of firms is arbitrary, so we think that the average payoff is a more relevant statistic, and second, as Ray and Vohra (1999) show, as δ converges to 1, the division of the surplus converges to equal division (Theorem 3.1).

2.4 Full Compatibility

We characterize the equilibrium coalition structure as the strength of the network effect varies. *A priori* we expect that, when the network effects are very strong, the market will tend towards full compatibility, that is, all firms will choose to be in the same platform. This is proved in Proposition 1, which shows that the unique equilibrium will be full compatibility for sufficiently strong network effects. Moreover, as the industry size increases, stronger and stronger network externalities are required to get full compatibility as equilibrium. The region of full compatibility is seen above the highest line in Figure 1.

All the Propositions, Lemma and Corollaries below have an extra qualification: there exists a $\delta^* \in (0, 1)$, such that, for all $\delta \in (\delta^*, 1)$, the propositions are true. In all the proofs, we first study the equilibria of game Δ. Then, we check that the payoffs are weakly decreasing weakly in a firm's index in game Δ. Thus, we extend the results to game $\Gamma(\delta)$ with $\delta \in (\delta^*, 1)$.

Proposition 1: *The (unique) equilibrium of game $\Gamma(\delta)$ with no-delay is full compatibility (i.e., $C = \{N\}$) if and only if $\alpha \geq \frac{2(N-1)}{N+2(N-1)}$, and $\lim_{N\to\infty} \frac{2(N-1)}{N+2(N-1)} = 2/3$.*

Proof: We start the proof by analyzing game Δ. We first prove the following lemma.

Lemma 1: *In any subgame of Δ, the first stage game with n firms left to be divided into coalitions, all n remaining firms will form a single coalition if and only if*

$$\alpha \geq \frac{2(n-1)}{N+2(n-1)}.$$

***Proof**:* The proof is by induction. Take $n = 2$. Given the division of firms up to this moment, the current firm makes a choice between two coalition structures that assign the last two firms as $\{1, 1\}$ or $\{2\}$. Direct computation shows that the second will chosen if and only if

$$\alpha\frac{2}{N}-(1-\alpha)\frac{4}{N^2}\geq\alpha\frac{1}{N}-(1-\alpha)\frac{2}{N^2}. \tag{3}$$

That is equivalent to

$$\alpha\geq\frac{2}{N+2}=\frac{2(n-1)}{N+2(n-1)}. \tag{4}$$

So the Lemma is true for $n=2$. Now suppose it is true for any $n = \{2, \ldots, k\}$. We now show that this implies that it holds for $n = k + 1$.

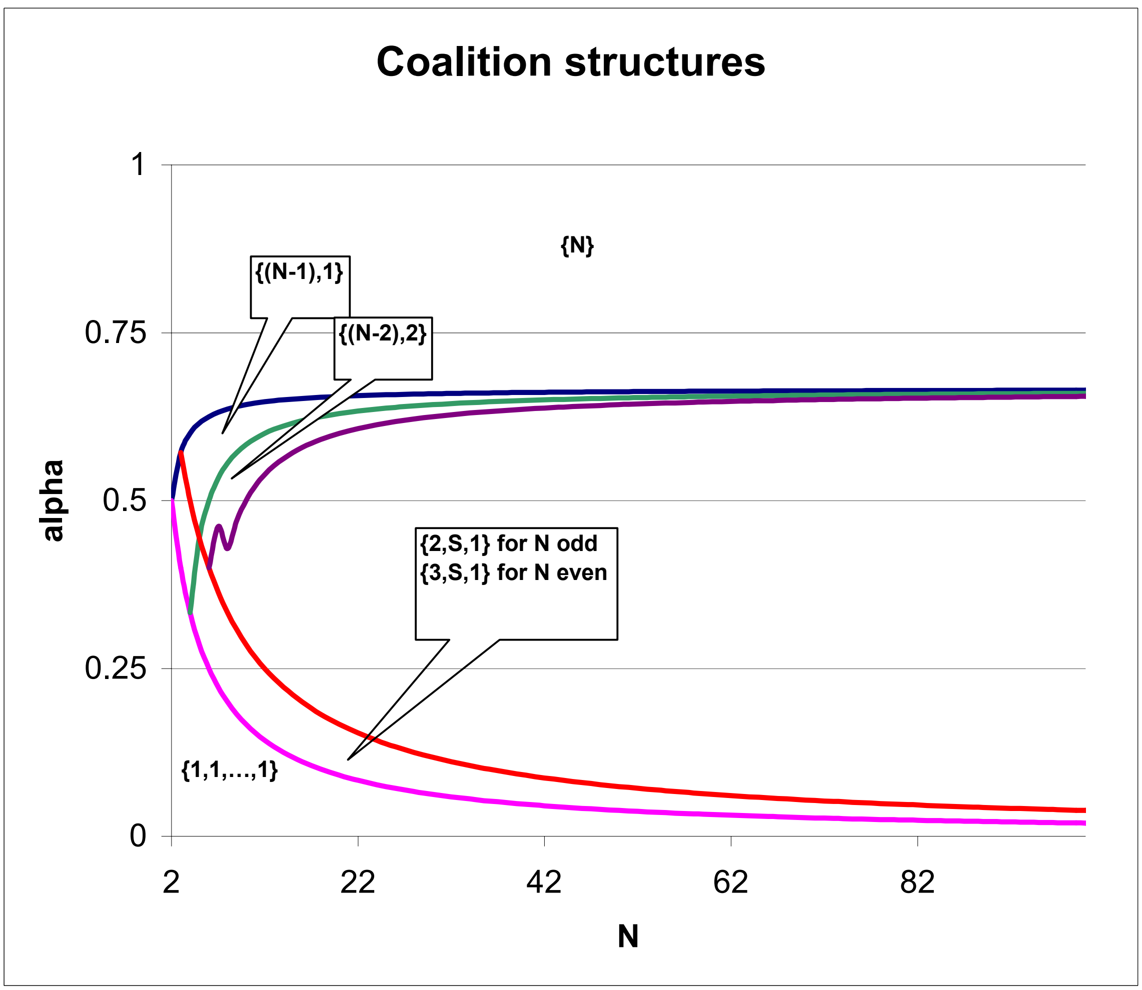

Figure 1

First, notice that $\frac{2(k-1)}{N+2(k-1)}$ is increasing in *k*. So if $\alpha \geq \frac{2(k-1)}{N+2(k-1)}$ and the current firm chooses to propose a coalition of smaller size than $k + 1$, the remaining firms will form exactly one coalition. So the firm choosing x_i for any coalition structure established in previous moves, induces a coalition structure $\{k + 1\}$ or $\{x_i, (k + 1 - x_i)\}$. This firm will choose a single coalition if and only if:

$$\alpha \frac{k+1}{N} - (1-\alpha)\frac{(k+1)^2}{N^2} \geq \alpha \frac{x_i}{N} - (1-\alpha)\left[\frac{(k+1-x_i)^2 + x_i^{\,2}}{N^2}\right]. \tag{5}$$

Call $y = k + 1 - x_i$ the number of firms that are left by firm *i*. Condition (5) is satisfied if and only if

$$\alpha \geq \frac{2(k+1-y)}{N+2(k+1-y)}. \tag{6}$$

The RHS is decreasing in y. So, if $\alpha \geq \frac{2(k)}{N+2(k)} = \frac{2(n-1)}{N+2(n-1)}$, the current firm chooses $x_i = k + 1$. If α is smaller than this cut-off, then even if the remaining firms form one coalition, firm 1 prefers some $x_i < k + 1$ over $k + 1$. Finally, if the firm prefers not to form one coalition from remaining firms when it expects the residual firms to form one coalition, it does so even more when it expects them to form more then one coalition (because this decreases the *HH(C)* index). So, for any $\alpha < \frac{2(n-1)}{N+2(n-1)}$, the firm will not choose $x_i = k + 1$. That finishes the proof of the lemma.

Now, by looking at the first subgame of the game Δ when $n = N$, we get full compatibility as the unique outcome. As the payoffs are trivially weakly decreasing for high discount factors there exists a no-delay equilibrium of $\Gamma(\delta)$, and it generates full compatibility. Finally, it is straightforward to check that the conditions of Theorem 3.4

in Ray and Vohra (1999) are satisfied so the no-delay full compatibility is the unique equilibrium outcome of $\Gamma(\delta)$ for high discount factors.

Also note that as N tends to infinity the lower bound of α in proposition 1 increases to 2/3. Thus, for any industry size, for strong network externalities, $\alpha \geq 2/3$, the equilibrium is full compatibility.

2.5 Total Incompatibility Equilibria

On the other extreme, we expect that, when network effects are weak, firms will choose mutually incompatible standards since there is less competition among mutually incompatible products. This is proved in Proposition 2, which shows that the unique equilibrium will be total incompatibility for sufficiently weak network effects. The region of total incompatibility is seen below the lowest line in Figure 1. The upper boundary of total incompatibility region tends to zero as the size of the industry tends to infinity. This means that, in an industry of a very large number of participants, even very small network effects will result in some technical standards coalition formation.

Proposition 2: *The (unique) equilibrium of game $\Gamma(\delta)$ is no-delay total incompatibility (i.e., $C = \{1, 1, \ldots, 1\}$ with N elements) if and only if $\alpha < \dfrac{2}{N+2}$, and $\lim_{N\to\infty} \dfrac{2}{N+2} = 0$.*

Proof: Start with the game Δ. Again, the proof is by induction. Take a subgame with $n = 2$ firms left. By Lemma 1, the unique outcome is full incompatibility if and only if $\alpha < \dfrac{2}{N+2}$. Now suppose that in any subgame with $n = \{1, \ldots, k\}$ the outcome is full incompatibility if and only if $\alpha < \dfrac{2}{N+2}$. We show that this implies full incompatibility in a subgame with $n = k + 1$ if and only if $\alpha < \dfrac{2}{N+2}$. In such a subgame, the firm making a proposal knows that when it selects any x firms to form a coalition with it, the remaining firms will form singleton coalitions (regardless of the coalition structure of firms that already formed coalitions). So this firm will decide to form a singleton coalition if and only if:

$$\alpha\frac{1}{N} - (1-\alpha)\frac{k+1}{N^2} > \alpha\frac{x}{N} - (1-\alpha)\frac{k+1-x+x^2}{N^2} \tag{7}$$

for every $x \in \{2, \ldots, (k+1)\}$. Equation (7) can be simplified to $\alpha < \dfrac{x}{N+x}$, which is true for every $x \in \{2, \ldots, (k+1)\}$ if and only if $\alpha < \dfrac{2}{N+2}$. This finishes the proof for any subgame, including the whole game. Trivially the payoffs are weakly monotonic, so there exists a no-delay equilibrium of $\Gamma(\delta)$. Also, trivially the condition of Theorem 3.4 in Ray and Vohra (1999) is satisfied. Thus, for high discount factors, this is a unique equilibrium outcome of $\Gamma(\delta)$.

2.6 Partial Compatibility Equilibria; Equilibria Close To Compatibility

We now investigate further the structure of the equilibrium for intermediate strengths of the network effects, where the equilibrium is neither full compatibility nor total incompatibility. When network externalities are relatively strong, the resulting equilibria are close to full compatibility. Proposition 3 shows that for strong network externalities which are slightly weaker that the strength that guarantees full compatibility, the equilibrium will have all but one firms in a large coalition. As the strength of the network externality decreases even more, Proposition 4 shows that the resulting equilibrium will have all but two firms in a large coalition, and the remaining two firms will form their own coalition.

Proposition 3: *If* $N \geq 4$ *and* $\alpha \in \left[\frac{2(N-3)}{N+2(N-3)}, \frac{2(N-1)}{N+2(N-1)}\right)$ *then the unique equilibrium coalition structure of game* $\Gamma(\delta)$ *is* $\{N - 1, 1\}$.

Proof: Again we start with the game Δ. First, when $\alpha \in \left[\frac{2(N-2)}{N+2(N-2)}, \frac{2(N-1)}{N+2(N-1)}\right)$, then from Lemma 1 we know that the equilibrium coalition structure will not be $\{N\}$ and for any $x < N$ chosen by firm 1, the remaining firms will form one coalition. So the first firm will choose $x = N - 1$ if and only if:

$$\alpha\frac{N-1}{N} - (1-\alpha)\frac{(N-1)^2+1}{N^2} \geq \alpha\frac{N-1-y}{N} - (1-\alpha)\frac{(N-1-y)^2+(1+y)^2}{N^2} \tag{8}$$

where $y = N - 1 - x$, and $y \in \{1, \ldots, (N - 2)\}$. Condition (8) can be rewritten as:

$$\alpha \geq \frac{2(N-y-2)}{N+2(N-y-2)}. \tag{9}$$

The RHS is decreasing in y and is at most $\frac{2(N-3)}{N+2(N-3)}$, so given our restriction on α the outcome is $\{N - 1, 1\}$. Now consider any $\alpha \in \left[\frac{2(N-3)}{N+2(N-3)}, \frac{2(N-2)}{N+2(N-2)}\right)$. If $x >$ 1, then the remaining $N - 2$ firms again form a single coalition. Given expression (9), we know then that the optimal choice from $x \in \{2, \ldots, N\}$ is $N - 1$. It remains to compare the profits of firm 1 in coalition structures {1, N - 2, 1} and {N - 1, 1}. The second one is chosen if and only if: $\alpha > \frac{2}{N+2} \geq \frac{2(N-3)}{N+2(N-3)}$.

As the sizes of coalitions are decreasing in the index of the proposing firm, the payoffs of the firms in them are decreasing as well. Thus, for high discount factors, *{N - 1, 1}* is a no delay equilibrium coalition structure of *Γ(δ)*. Also, the technical sufficient condition of Theorem 3.4 of Ray and Vohra is satisfied, what provides uniqueness. This finishes the proof.

Proposition 4: *If N ≥ 8 and* $\alpha \in \left[\frac{2(N-5)}{N+2(N-5)}, \frac{2(N-3)}{N+2(N-3)}\right)$*, then there exists a no-delay equilibrium of game Γ(δ) with equilibrium coalition structure {N - 2, 2}. For N = {6, 7} the same is true for* $\alpha \in \left[\frac{2(N-4)}{N+2(N-4)}, \frac{2(N-3)}{N+2(N-3)}\right)$.

Proof*: See Appendix.*

2.7 Partial Compatibility Equilibria; Equilibria Close to Incompatibility

When network effects are weak, coalitions tend to be small since the benefits from joining an association are small. When the strength of the network effects is very small, we have seen in Proposition 2 that there is total incompatibility at equilibrium. For immediately higher network effects, Proposition 5 shows that the equilibrium has a singleton coalition, a number of coalitions of size two, and, for odd industry size, one coalition of size 3. Thus, we can generally say that coalition sizes are very small for weak network externalities.

Proposition 5: *If* $\alpha \in \left[\frac{2}{N+2}, \frac{4}{N+4}\right)$, *then there exists a no-delay equilibrium in game Γ(δ) with equilibrium coalition structure* $C = \{2, S, 1\}$ *for* $N \geq 3$ *if* N *is odd and* $C = \{3, S, 1\}$ *if* N *is even, where* S *is a vector of 2s with* $(N - 3)/2$ *elements when* N *is odd and with* $(N - 4)/2$ *elements when* N *is even.*

Proof*: See Appendix.*

2.8 Inequality Of Coalition Sizes At Equilibrium

One of the key results of our paper is the existence of very significant inequalities among the sizes of coalitions at equilibrium for most parameters. We show below that, except for the parameters that lead to total incompatibility, there are very significant inequalities among the sizes of coalitions at equilibrium. Specifically, Proposition 6 shows that, when there are two coalitions at equilibrium, the smaller coalition is smaller that half the size of the larger coalition plus 1. Corollary A of Proposition 6 shows the same inequality between the smallest and the largest coalitions in the case of many coalitions. Corollary B of Proposition 6 shows that there are no coalition structures with all coalitions being of equal size except for the coalition structure of total incompatibility, that is, there are no equilibrium coalition structures with equal size coalitions size for all $\alpha \geq 2/(2 + N)$ (from Proposition 2).

Proposition 6 (asymmetry of coalitions at equilibrium): *Consider any* α *such that there are exactly two coalitions formed in a no-delay equilibrium of game Γ(δ). Let their sizes be* $n_1 \geq n_2$. *It is always true that* $n_2 < 1 + n_1/2$.

Proof: First, by Lemma 1 the smaller coalition has to be formed second. Otherwise firm 1 should deviate and choose $x_1 = n_2$. After such a move, the remaining firms would still form one coalition and hence firm 1 would be better off. Second, again by Lemma 1, given that the last n_2 firms form one coalition, it has to be the case that

$$\alpha \geq \frac{2(n_2 - 1)}{N + 2(n_2 - 1)} . \tag{10}$$

In that case firm 1 knows that if it chooses $x_1 = n_1 + 1$ instead of $x_1 = n_1$, the remaining firms will still form one coalition. It prefers to choose n_1 if the following condition holds

$$\alpha \frac{n_1}{N} - (1-\alpha)(\frac{n_1{}^2 + n_2{}^2}{N^2}) > \alpha \frac{n_1 + 1}{N} - (1-\alpha)(\frac{(n_1+1)^2 + (n_2-1)^2}{N^2}) . \tag{11}$$

This is true if

$$\alpha < \frac{2(n_1 - n_2 + 1)}{N + 2(n_1 - n_2 + 1)} . \tag{12}$$

Conditions (10) and (12) have a non-empty intersection if and only if $n_1 > 2(n_2 - 1)$, *i.e.*, $n_2 < 1 + n_1/2$. This finishes the proof. This result also holds for equilibria with delay if their coalition structures correspond to the coalition structure of game Δ.

Corollary A to Proposition 6: *Consider any no-delay equilibrium coalition structure of game $\Gamma(\delta)$ with at least 2 coalitions. Denote the size of the largest coalition by n_1 and the size of the smallest coalition by n_2. It is always true that $n_2 < 1 + n_1/2$.*

Proof: Take the last two coalitions formed and denote their sizes by y_1 and y_2. Clearly $y_1 \leq n_1$ and $y_2 \geq n_2$. Reasoning the same way as in Proposition 5 we get $y_1 > 2(y_2 - 1)$. (This result also holds for equilibria with delay if their coalition structures correspond to the coalition structure of game Δ.)

Corollary B to Proposition 6: *For any* $n > 1$*, the egalitarian coalition structure* $\{n, n, \ldots, n\}$ *is never a no-delay equilibrium of* $\Gamma(\delta)$ *and never a delay equilibrium with the equilibrium structure as in game* Δ*.*

Proof: From Corollary A to Proposition 6, if $\{n, n, \ldots, n\}$ is an equilibrium, then $n < 1 + n/2$ which is equivalent to $n < 2$.

A number of other results are also interesting. First, for any $N<14$,[10] it is *not* the case that the equilibrium *HH(C)* index is weakly monotone in α. For example, for $N = 10$ for $\alpha=0.37$ the equilibrium coalition structure is {6,3,1} and for $\alpha=0.4$ the equilibrium coalition structure is {5,4,1}, and the corresponding *HH(C)* are 0.46 and 0.42. Second, in this range of *N* it is the case that the equilibrium number of coalitions is weakly monotone in α.

2.9 Other Features Of The Equilibria

Figure 2 illustrates the relationship between the concentration index, the number of coalitions, and the ratio of firms in the largest to the second largest coalition as the strength of the network externalities varies. These diagrams were made for N = 13 firms. Table 1 shows the corresponding equilibrium coalition structures.

[10] In this range we obtain unique stationary no-delay equilibria in pure strategies – for every N and α - see section 2.10 - so we can talk about the equilibria in the whole range of α. For other N in general the equilibria of the game Δ do not always correspond to equilibria of the game Γ so we cannot present the full comparative statics.

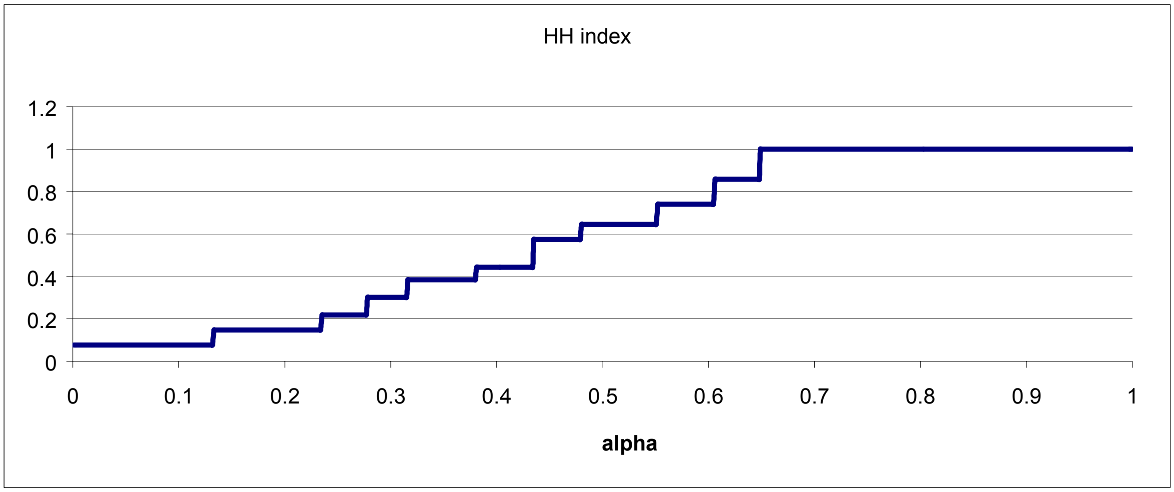

Figure 2(a)

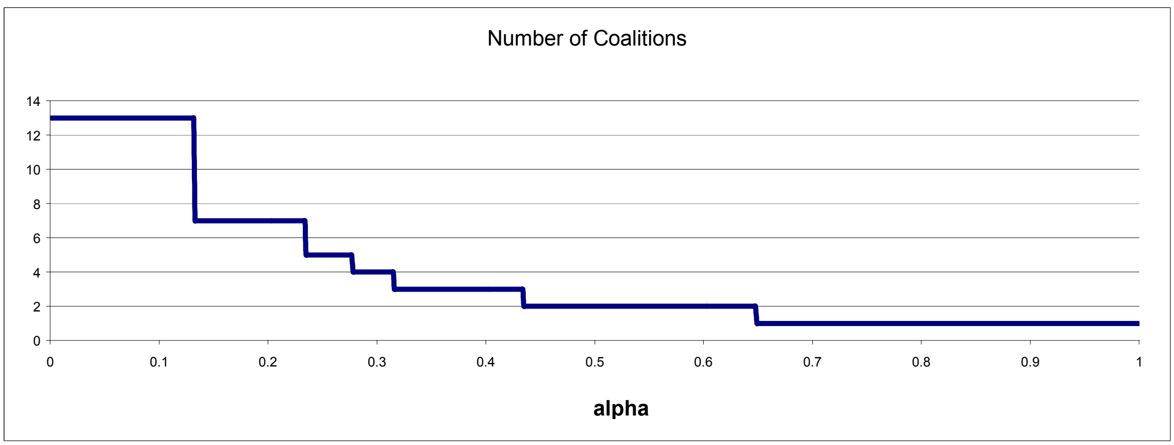

Figure 2(b)

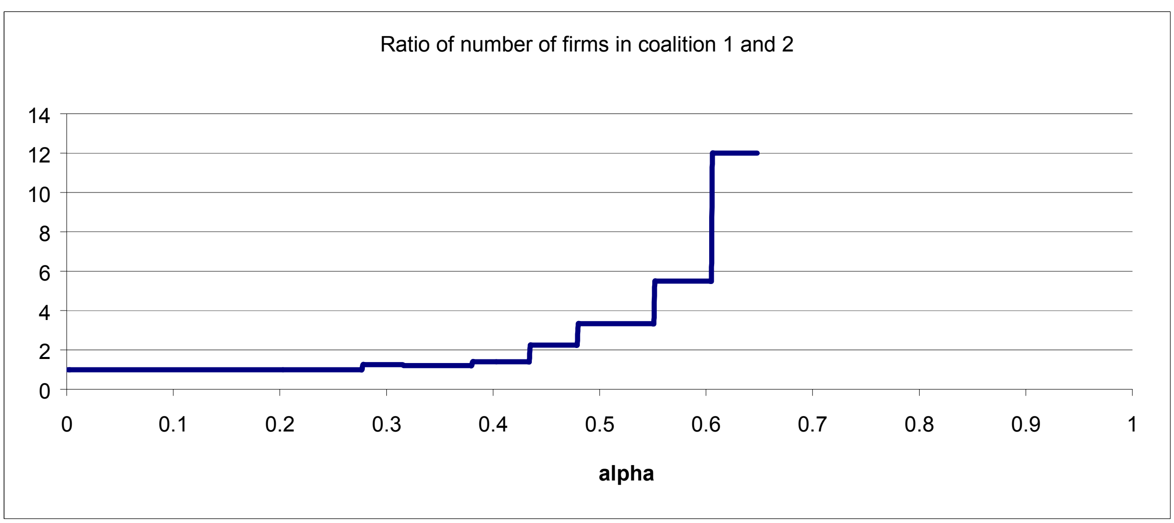

Figure 2(c)

Table 1	
N = 13	
α	**Equilibrium coalition structure**
0	{1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1}
0.1333	{2, 2, 2, 2, 2, 2, 1}
0.2353	{3, 3, 3, 3, 1}
0.2778	{5, 4, 3, 1}
0.3158	{6, 5, 2}
0.381	{7, 5, 1}
0.4348	{9, 4}
0.48	{10, 3}
0.5517	{11, 2}
0.6061	{12, 1}
0.6486	{13}

As α decreases, the equilibrium moves from full compatibility to partial incompatibility with one firm splitting off, and then two , as shown in Propositions 1, 3, and 5. The example shows that this pattern continues for lower α, with the smaller coalition increasing to three and then four members. Then, in the three coalitions equilibria, the middle size coalition does not change as the largest coalition shrinks and the smallest increases.

2.10 Example Of Lack Of Existence Of A Pure Strategies No Delay Equilibrium

We now discuss an example of a case when game Γ does not have a pure strategy, no-delay equilibrium. The smallest N for which it happens is N = 14. For N = 14, this is the case for $\alpha \in (0.3, 0.3226)$. We fix $\alpha = 0.31$. We first analyze the game Δ. First, from Lemma 1 we have that, when there are $n \le 4$ firms left in any subgame, they will form one coalition. Calculating the payoffs for all other n (the number of firms left in the subgame), we show in Table 2 the structure in the rest of the game. Thus, the number of firms per coalition is not monotonic in the whole game. To see why, consider the different possibilities that the first firm chooses a coalition of size x, $x \in \{1, 2, \ldots, 14\}$, in Table 3.

Table 2

n	**Coalition structure C**
2	{2}
3	{3}
4	{4}
5	{4, 1}
6	{5, 1}
7	{5, 2}
8	{6, 2}
9	{6, 3}
10	{5, 4, 1}
11	{6, 4, 1}
12	{7, 4, 1}
13	{6, 5, 2}
14	{4, 5, 4, 1}

Table 3

x	**Remaining Firms**	**Coalition structure C**	**$V_1(C)$**
1	13	{1, 6, 5, 2}	k-0.2102
2	12	{2, 7, 4, 1}	k-0.20214
3	11	{3, 6, 4, 1}	k-0.15184
4	10	{4, 5, 4, 1}	k-0.11561
5	9	{5, 6, 3}	k-0.13571
6	8	{6, 6, 2}	k-0.13469
7	7	{7, 5, 2}	k-0.11959
8	6	{8, 5, 1}	k-0.13969
9	5	{9, 4, 1}	k-0.14571
10	4	{10, 4}	k-0.18694
11	3	{11, 3}	k-0.21408
12	2	{12, 2}	k-0.25531
13	1	{13, 1}	k-0.31061
14	0	{14}	k-0.38

We see that firm 1 obtains the highest value $V_1(C)$ by selecting $x = 4$, and that leads to coalition structure {4, 5, 4, 1}. This means that the firms in the second coalition have higher profits than the firms in the first one and hence game Γ (for N = 14 and α = 0.31) does not have a pure strategies no delay equilibrium. Notice that the reason we get this example is that, if firm 1 chooses x = 4, the remaining firms form 3 coalitions. However, any x > 4 results in fewer number of coalitions, what given the network effect is weak, decreases profits of firm 1. We can also show that, in this case, there is also no mixed strategy no delay equilibrium.[11]

3. Concluding Remarks

We analyze the formation of technical standards coalitions in markets with network effects. We find that the extent and size of coalitions at equilibrium depends crucially on the degree of the intensity of network effects. When network effects are very strong, full compatibility prevails. When externalities are slightly weaker, two standards coalitions are formed, a singleton, and one with all remaining firms. On the other extreme, for very weak network effects, the equilibrium is total incompatibility, and for

[11] But even a stronger result can be proven: in this case there is also no mixed strategy no-delay equilibrium. First, suppose that such equilibrium exists. Then it must be the case that the equilibrium coalition structure is {4, 5, 4, 1}. Second, the payoff of a player in the coalitions of size 4 is – 0.116, the payoff of a player in the coalition of size 5 is –0.093 and the payoff of the singleton player is –0.182. Third, there must exist two players i and j such that when it is i to make an offer with all 14 players left in the game, the probability that player j will end up in the coalition with 5 players is strictly positive and at least 5 times the probability that he will end up in the singleton coalition. This reasoning is through averages: on average it is true as there are 5:1 players in those two coalitions. If this is true for the average, the weak inequality has to be true for at least one player.

Now consider the strategy of player j. If he moves first, in the no-delay equilibrium he is supposed to get payoff –0.116. If he deviates and makes an unacceptable offer to player i, he can expect a payoff at least $\delta((1 - p)(-0.116) + p((5/6)(-0.093) + (1/6)(-0.182))) = \delta((1 - p)(-0.116) + p(-0.108))$, where $p > 0$ is the probability that he will end up in either coalition of size 5 or of size 1. Clearly for any $p > 0$ there exists $\delta < 1$ such that this deviation is profitable. So there does not exist any no-delay equilibrium of game $\Gamma(\delta)$ in stationary strategies.

slightly more intense network effects, coalitions are of small size. We characterize a number of other equilibria for intermediate strengths of network externalities.

4. References

Bloch, F. (1995). "Endogenous Structures of Association in Oligopolies," *Rand Journal of Economics,* vol. 26, pp. 537–556.

Bloch, F. (1996), "Sequential Formation Of Coalitions In Games With Externalities And Fixed Payoff Division," *Games and Economic Behavior*, vol. 14, pp. 90–123.

Chatterjee, K., B. Dutta, D. Ray, and K. Sengupta (1993). "A Noncooperative Theory of Coalitional Bargaining," *Review of Economics Studies,* vol. 60, pp. 463–477.

Economides, N. (1985), "Equilibrium Coalition Structures," Discussion Paper No. 273, Columbia University.

Economides, N., and F. Flyer (1998), "Compatibility and Market Structure for Network Goods," Discussion Paper EC-98-02, Stern School of Business, N.Y.U.

Ray, D., and R. Vohra (1999), "A Theory of Endogenous Coalition Structures," *Games and Economic Behavior*, vol. 26, pp.286-336.

Ray, D., and R. Vohra (1997), "Equilibrium Binding Agreements," *Journal Of Economic Theory*, vol. 73, pp. 30-78.

Rubinstein, A. (1982)."Perfect Equilibrium in a Bargaining Model," *Econometrica,* vol. 50, pp. 97–109.

Selten, R. (1981). "A Noncooperative Model of Characteristic Function Bargaining" in *Essays in Game Theory and Mathematical Economics in Honor of Oskar Morgenstern* (V. Bohm, and H. Nachtkamp, Eds.), pp. 131–151. Mannheim: Bibliographisches Institut Mannheim.

5. <u>Appendix</u>

Proof of Proposition 4: First we show the result for $N \geq 6$. Consider any subgame with n firms left and with $\alpha \in \left[\frac{2(n-4)}{N+2(n-4)}, \frac{2(n-3)}{N+2(n-3)}\right)$. The firm making current proposal x can induce one of the following 3 coalition structures of the remaining firms: {1, n - 2, 1}, {2, n - 3, 1}, {x, n - x} (we know this using from the previous propositions, we also know that $x = n$ is not an optimal choice).

Now, choosing $x = n - 2$ dominates $x = 1$ if

$$\alpha \frac{n-3}{N} - (1-\alpha)\frac{(n-2)^2 + 2^2 - 2 - (n-2)^2}{N^2} \geq 0 .$$

This is the case when $\alpha \geq \frac{z}{N+z}$ where $z = \frac{2}{n-3}$. So this is the case when $2(n-4) \geq \frac{2}{n-3}$ and is true for all $N \geq n \geq 5$. Choosing $x = n - 2$ dominates $x = 2$ if

$$\alpha \frac{n-4}{N} - (1-\alpha)\frac{(n-2)^2 + 2^2 - 2^2 - (n-3)^2 - 1}{N^2} \geq 0 .$$

This is the case when $\alpha \geq \frac{z}{N+z}$ where $z = \frac{2(n-3)}{n-4}$. It is satisfied when $2(n-4) \geq \frac{2(n-3)}{n-4}$ and is true for all $N \geq n \geq 6$.

Choosing $x = n - 2$ dominates any $n - 2 \geq x \geq 3$ if

$$\alpha \frac{n-2-x}{N} - (1-\alpha)\frac{(n-2)^2 + 2^2 - x^2 - (n-x)^2}{N^2} \geq 0 .$$

This is the case when $\alpha \geq \frac{z}{N+z}$, where $z = \frac{-4n + 2nx - 2x^2 + 8}{n-2-x}$. It is satisfied when $2(n-4) \geq z$. For any $x \leq n - 2$, we can show that it is true by showing that

$2(n-4)(n-2-x)+4n-2nx+2x^2-8$ is minimized for $x = n - 2$ (and then $z = 2(n - 4)$).

For $x = n - 1$ the condition is $\alpha\frac{1}{N}-(1-\alpha)\frac{(n-1)^2+1-2^2-(n-2)^2}{N^2}<0$ or

$\alpha<\frac{2(n-3)}{N+2(n-3)}$, which is the case. So we have proved the proposition for $N \geq 6$ and

$\alpha\in\left[\frac{2(N-4)}{N+2(N-4)},\frac{2(N-3)}{N+2(N-3)}\right)$.

Now consider $\alpha\in\left[\frac{2(n-5)}{N+2(n-5)},\frac{2(n-4)}{N+2(n-4)}\right)$ and a subgame with $n \geq 8$ firms left. The current choice leads to one of the following coalition structures:

$\{1, n - 3, 2\}$, $\{2, n - 3, 1\}$, $\{3, n - 4, 1\}$, and $\{x, n - x\}$ for $x \geq 4$. First, clearly, choosing $x = 2$ dominates $x = 1$. Second choosing $x = n - 2$ dominates $x = 2$ if, as above,

$\alpha\geq\frac{z}{N+z}$ where $z=\frac{2(n-3)}{n-4}$. It is true when $2(n-5)\geq\frac{2(n-3)}{n-4}$ and is true for all $N \geq n \geq 7$. Third, choosing $x = n - 2$ dominates $x = 3$ if

$$\alpha\frac{n-5}{N}-(1-\alpha)\frac{(n-2)^2+2^2-3^2-(n-4)^2-1}{N^2}\geq 0,$$

which is true if $\alpha\geq\frac{z}{N+z}$ where $z=\frac{4n-18}{n-5}$. This is true when $2(n-5)\geq\frac{4n-18}{n-5}$ and for all $N\geq n\geq 8$.

Fourth, choosing $x = n - 2$ dominates any $n-2\geq x\geq 4$ if

$$\alpha\frac{n-2-x}{N}-(1-\alpha)\frac{(n-2)^2+2^2-x^2-(n-x)^2}{N^2}\geq 0.$$

This is the case when $\alpha\geq\frac{z}{N+z}$, where $z=\frac{-4n+2nx-2x^2+8}{n-2-x}$. It is satisfied when $2(n-5)\geq z$. For any $x \leq n - 2$ we can show that it is true by showing that

$2(n-5)(n-2-x)+4n-2nx+2x^2-8$ is minimized for x = n - 2.5. Checking at $x = n$ - 3 is thus sufficient and indeed 2(n - 5) = z for this x. For $x = n$ - 1 the condition remains the same as above and is still satisfied. This finishes the proof of the proposition.

Proof of Proposition 5: It can be easily verified that given the restriction on α in any subgame with:

a) n = 2 firms left, the remaining firms form structure {2};

b) n = 3 firms left, the remaining firms form structure {2, 1};

c) n = 4 firms left, the remaining firms form structure {3, 1}; and

d) n = 5 firms left, the remaining firms form structure {2, 2, 1}.

So the Proposition is true for N = {3, 4}. Now suppose that, in any subgame with $n = k$ firms, the remaining firms form a coalition structure described above. We show that it implies that it is also the case for $n = k$ +1.

From Proposition 1, we know that, for $N \geq 3$, the equilibrium structure is not full compatibility. When the current firm that makes a proposal picks x, it leads to a coalition structure {x, 3, 2, …, 2, 1} if $(k+1-x)$ is even and to structure {x, 2, 2, …, 2, 1} if $(k+1-x)$ is odd. We start by showing that x = 2 dominates all other even x, and x = 3 dominates all other odd x. Any even x such that k - 1 > x > 2 dominates x = 2 if and only if

$$\alpha\frac{x-2}{N}-(1-\alpha)(\frac{x^2-2x}{N^2})\geq 0.$$

That is equivalent to:

$$\alpha \geq \frac{x}{N+x},$$

which is not the case for any even x > 2.

Any odd x such that k – 1 > x >1 dominates x = 1 if and only if:

$$\alpha \frac{x-1}{N} - (1-\alpha)(\frac{x^2 - 2(x-1) - 1}{N^2}) \geq 0.$$

That is equivalent to:

$$\alpha \geq \frac{x-1}{N+x-1},$$

and is satisfied only for $x = 3$. So given our restrictions on $\alpha, x = 3$ dominates all other odd *x.*

The remaining possibilities for x are {(k - 1), (k)} and they lead to coalition structures of the remaining firms: {k - 1, 2} and {k, 1} respectively. Using similar arguments as above it can be shown that in these two special cases $x = 2$ again dominates any other even x and $x = 3$ again dominates any other odd x (in these special cases we no longer have to check $x = 1$).

The final step is to consider when selecting $x = 2$ dominates $x = 3$. When $n = k + 1 > 5$ is odd then choosing $x = 2$ leads to a structure {2, 2, 2, S, 1} and choosing $x = 3$ leads to a structure {3, 3, S, 1}. The second is chosen if and only if:

$$\alpha \frac{1}{N} - (1-\alpha)(\frac{9+9-3*4}{N^2}) \geq 0 \text{, } i.e., \ \alpha \geq \frac{6}{N+6},$$

which is always true. Therefore, {2, S, 1} is the equilibrium structure in this subgame. When $n = k + 1 > 4$ is even then choosing $x = 2$ leads to a structure {2, 3, S, 1} and choosing x=3 leads to a structure {3, 2, S, 1}. Clearly the second one always generates better payoffs for the firm currently making the offer, so {3, S, 1} is the equilibrium structure in this subgame. Finally, taking the first subgame of the whole game, we obtain the Proposition.